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New Model for Vortex Decay in the Atmosphere

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A new vortex decay model for the prediction of the descent of aircraft trailing vortices subjected to realistic environmental conditions (stratification, turbulence, crosswind, headwind, shear effects, and ground effect) is presented, and the model is applied to field data obtained with Lidar in Memphis and Dallas–Fort Worth airports. Although the model has not yet been fully optimized, the predictions and field data compare reasonably well. Some flights, particularly in unstable environments, exhibit behavior unexplainable in terms of the assumed, measured, and/or indirectly calculated input parameters, for example, vortex separation, uncertainties in Lidar measurements, stratification, shear, gravity currents, head- and crosswinds, turbulent kinetic energy, and/or the eddy dissipation rate.

Introduction

THE current impetus for research on single and mutually straining vortices comes, in part, from the need to enhance the capacity of large airports by reducing wake-hazard-imposed aircraft separations for various flight modes and meteorological conditions. The review of the extensive literature[1] indicates that there are major obstacles to the understanding of the physics of trailing vortices for the prediction of their transport and decay, for the promotion of their receptivity to instability, and for the rapid reduction and spreading of their vorticity into random turbulence. The interaction of vortices with what surrounds them and the relation between the full-scale flight tests and the physical/numerical laminar-flow experiments remain elusive. It is rather unfortunate that all of the governing decay parameters cannot be realized in small-scale laboratory experiments and direct numerical simulations. Even the highest Reynolds numbers ($Rec = \Gamma/\nu$, where Γ is the circulation and ν the kinematic viscosity) reached in wind tunnels or towing basins (typically, $Rec = 4 \times 10^4$) are several orders of magnitude lower than what is possible for an aircraft, for example, $Rec = 3.6 \times 10^7$ for a DC-10-30. In fact, the Reynolds number is often not high enough to achieve a well-developed inertial subrange.

It has been clear for quite some time that stratification, shear, turbulence, and instabilities from all sources[2] with their direct or indirect consequences are the fundamental demise mechanisms that destroy the coherence of a vortex wake. The ambient turbulence has a strong influence on the stability and on the gradual and/or catastrophic demise of vortices.[3–6] However, preexisting turbulence is extremely difficult to quantify, particularly before each flight.[7] The physics of the decay mechanism resulting from the interaction of the ambient turbulence with the vortex is not understood, it is only inferred from numerical simulations[6,8–12] and the migration and lifespan of vortices in small-scale experiments[3,4] conducted in grid-generated turbulence.

It is in view of the foregoing that NASA[13,14] has undertaken to develop an Aircraft Vortex Spacing System (AVOSS), as an element of the terminal area productivity program, that led to field experiments[13] at several international airports [Idaho Falls, Memphis, John F. Kennedy, Dallas–Fort Worth (DFW), and Norfolk]. The ultimate objective of the AVOSS is to estimate as accurately as possible the length of time that a pair of trailing vortices will remain a hazard to any aircraft flying toward them.

The analysis of the representative field data has shown that the effect of the molecular diffusion on the downwash and demise of trailing vortices is inconsequential and inconsistent with the high Reynolds number field data[5] and that the vortex core is not a benign solid body in rotation. There is an intermittent exchange of mass, momentum, and vorticity across the core boundary. Large flow structures outside the core are stretched by velocity gradients induced by the vortex pair.[11,12] The vorticity transferred from the vortex to the ambient turbulence by the stretching process eventually reduces to smaller scales by turbulent diffusion.

Appreciation of these facts together with the current field data representing the state of the art is of special importance in assessing the results presented herein. They deal only with the data that gave rise to them, not with the great controversies surrounding the aircraft wakes.

Vortex Decay Models

Greene's Model

In 1986, Greene[16] proposed a model in which the rate of change of impulse per unit length of the vortex wake is equated to the sum of a viscous drag force (supposed to be acting on an idealized Kelvin oval), the buoyancy force due to stratification (accounting for both the temperature and humidity gradients), and a turbulence-generated viscous force. This led to the following differential equation:

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}T^2} + \frac{C_D L}{4\pi b}\left(\frac{\mathrm{d}Z}{\mathrm{d}t}\right)^2 + 0.82 Q^*\left(\frac{\mathrm{d}Z}{\mathrm{d}T}\right) + \frac{A N^{*2}}{2\pi b^2} Z = 0 \qquad (1)$$

in which $Q^* = \sqrt{(2\,\mathrm{TKE})}/V_0$, $V_0 = \Gamma_0/2\pi b$, $T = V_0 t/b$, $Z = z/b$, $V = \mathrm{d}Z/\mathrm{d}T$, A is the area of the wake oval, b is the vortex spacing, C_D is the drag coefficient, L is the width of the wake oval ($2.09b$), N^* is the normalized stratification parameter ($= Nb/V_0$) where N is the Brunt–Vaisala frequency, t is the time, TKE is the turbulent kinetic energy, V_0 is the initial wake descent speed, z is the vertical elevation, and Γ_0 is the initial vortex circulation, determined from the reported aircraft type, weight, and speed.

The original form of the model, as outlined, did not deal with ground effects, shear, headwind, and TKE profiles. Greene[16] carried out a parametric study to assess the effects of the model parameters. However, the predictions of the model have not been tested in light of any field data. Recently, Sarpkaya[15] has undertaken such a study of Greene's model using the field data obtained at Memphis. A similar comparison was undertaken by Robins et al.[8] with modifications to include the effects of vertical profiles of atmospheric stratification, turbulence, crosswind, and ground proximity. Note that the use of a drag force assumed to be acting on a free vortex pair is not hydrodynamically sound. Furthermore, the third term in Eq. (1), expressing the effect of turbulence, is based on an approximate analysis proposed by Donaldson and Blanin[9] about 25 years ago. The adjustment of the coefficient of the turbulence term (0.82) and the selection of a suitable drag coefficient C_D to make the predictions agree with the field data leave much to be desired.

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New Model

The rate of change of impulse per unit length of the vortex wake is equated to the sum of the buoyancy force due to stratification (accounting for both the temperature and humidity gradients) and the force due to the rate of change of circulation. The viscous drag force in Greene's model is eliminated because as noted, it is not hydrodynamically defensible. Furthermore, the nature of the rate of change of circulation is neither specified nor related to TKE. The resulting equation is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t} = 2\pi\rho b^2 \frac{\mathrm{d}v}{\mathrm{d}t} = -\rho A N^2 z - \rho b \frac{\mathrm{d}\Gamma}{\mathrm{d}t} \tag{2}$$

It is then hypothesized that the decay of the vortex pair is dictated by $\exp(-CT/T^*)$, where C is a constant and T^* is the time at which a catastrophic demise event (e.g., Crow[5] instability, core bursting) takes place. In other words, the intensity of the environmental turbulence determines T^*, and T^* in turn determines the dissipation rate. Meanwhile, the vortex pair continues to decay up to and beyond the said event. Formalizing this concept, we have

$$\Gamma/\Gamma_0 = \exp[-(C/T^*)T] = \exp(-MT) \tag{3}$$

where $M = C/T^*$. The justification for the proposed decay hypothesis rests with the experience gained from the analysis[15] of the Memphis and DFW field data, heuristic reasoning, and mathematical simplicity. We hope that a model having these features will also be robust and not overly sensitive to the uncertainties of the parameters specifying the atmospheric conditions; will apply equally well to all aircraft, stratification, and ambient turbulence; and will not require additional empirical constants. Its dependence on the eddy dissipation rate ε^* for the ambient turbulence is regarded as particularly significant because ε^* is one of the most fundamental parameters of turbulence, particularly for flows capable of achieving a well-developed inertial subrange.

Using Eqs. (2) and (3), one has

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}T^2} + \omega^2 N^{*2} Z + M\exp(-MT) = 0 \tag{4}$$

For $N^* = \text{const}$, no wind, no shear, no ground effects, and constant b, Eq. (4) has an exact solution given by

$$Z = [1/(M^2 + \omega^2 N^{*2})](\omega N^* \sin \omega N^* T - M e^{-MT} + M) \tag{5}$$

that satisfies the appropriate boundary conditions ($T = 0$, $Z = 0$ and $\mathrm{d}Z/\mathrm{d}T = 1$) and requires the selection of one constant (C) and the specification of the ambient turbulence in terms of $T^*(\varepsilon^*)$. These will be taken up later.

The solution of more complex cases, where there are lateral wind and shear effects and where the variation of ε^* and N^* with Z cannot be ignored, Eq. (4) needs to be solved numerically in a relatively short time so that the information is available to AVOSS on a real-time basis. This, in turn, requires sound parametric relationships between T^*, ε^*, and Z. Most of the ambient turbulence and TKE information comes from towers about 40 m high.[7] Although, it is theoretically possible to obtain an estimate of the spectral energy density (using tower measurements of 5- and 40-m data) and thereby calculate the dissipation rate directly, one obtains only an average over a height of 40 m. What is needed is the eddy dissipation from near ground level to several hundreds of meters up to provide reliable input to the numerical simulations.

The turbulence parameter $\varepsilon^*[=(\varepsilon b)^{1/3}/V_0]$, defined by Crow and Bate[6] with $V_0 = \Gamma/2\pi b$, was subsequently used by Tombach[3] and Sarpkaya and Daly[4] in their work on trailing vortices. Figure 1 shows two sets of experimental data,[3,4] two analytical predictions based on the original[6] and the modified[15] Crow and Bate model, and the results of the large eddy simulations (lifetimes to linking) by Han et al.,[14] all for a nonstratified medium. In more complex environments, the indirect effects of N^* on T^* also need to be taken into account.[2] Moreover, there are a number of issues regarding the variation of the eddy-dissipation rate and the indirect effects of stable

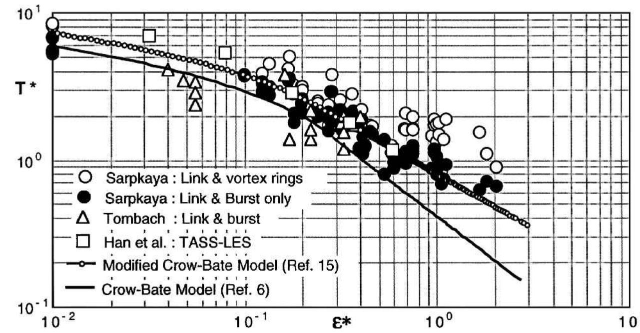


Fig. 1 T^* vs ε^*: models and physical and numerical experiments.

and unstable stratification on the inception of one or more catastrophic demise events that must be resolved before the proposed model can be used with confidence by AVOSS on a real-time basis. In weak turbulence (i.e., for ε^* less than about 0.02 or for ε less than about 0.03 cm^2/s^3 for a DC-10 with $\Gamma = 550$ m^2/s and $b = 37$ m), multiple linking (with an average wavelength of about $7.8b$) and the subsequent instability events destroy the coherence of vortices. In medium turbulence (i.e., for ε^* greater than about 0.02 and less than about 0.2, or for ε less than about 30 cm^2/s^3 for a DC-10), the dominant form of instability is the Crow instability (with decreasing wavelengths and integral length scales) and occasional vortex bursting. The physics of the latter is not understood but it is known that bursts do not lead to reconnection. Finally, for stronger turbulence, that is, for ε^* larger than about 0.3, vortex bursting, all other forms of instability, strong mixing in the overlapping region of the vortices, rollup of vortices about each other, and lateral displacements spread the vorticity irreversibly over a large area.[2–4]

Figure 1 shows that the original Crow and Bate[6] model serves as a lower bound to the link/burst data. It becomes inadequate for $\varepsilon^* > 0.2$ where the predicted decay is quite steep. The reasons lie in the highly restrictive nature of some of the assumptions made in their pioneering analysis. The most important ones are as follows: 1) "The atmospheric turbulence is regarded as independent of the vortices."[6] In other words, the distortion of the ambient turbulence by the trailing vortices and departures from statistical homogeneity and isotropy are not taken into account. 2) "The lifespan is determined by extrapolating linear theory to times when the displacement perturbations are comparable to the original vortex separations b."[6] 3) "The atmospheric turbulence is assumed to be steady in coordinates moving downward with the vortices."[6] As noted, both the wavelength and the integral length scale decrease with increasing ε^*, at least in small-scale experiments. Crow and Bate[6] drew attention to the situation wherein "There seems to be no straightforward way to handle such departures, and the assumption that they are unimportant will have to be tested by comparing the calculated lifespans to experiment." It is in accordance with their suggestion that the Crow and Bate[6] model has been rederived[15] to allow for the variation of the wavelength and the integral length scale. The resulting vortex lifespans in various intervals shown in Fig. 1 are represented by

$$\varepsilon^* T^{*\frac{4}{3}} = 0.7475, \qquad T^* < 2.25 \quad \text{or} \quad \varepsilon^* > 0.2535 \tag{6a}$$

$$\varepsilon^* = T^{*\frac{1}{4}}\exp(-0.70T^*),$$

$$2.256 < T^* < 7 \quad \text{or} \quad 0.0121 < \varepsilon^* < 0.2535 \tag{6b}$$

$$T^* = -180\varepsilon^* + 9.18,$$

$$7 < T^* < 9 \quad \text{or} \quad 0.001 < \varepsilon^* < 0.0121 \tag{6c}$$

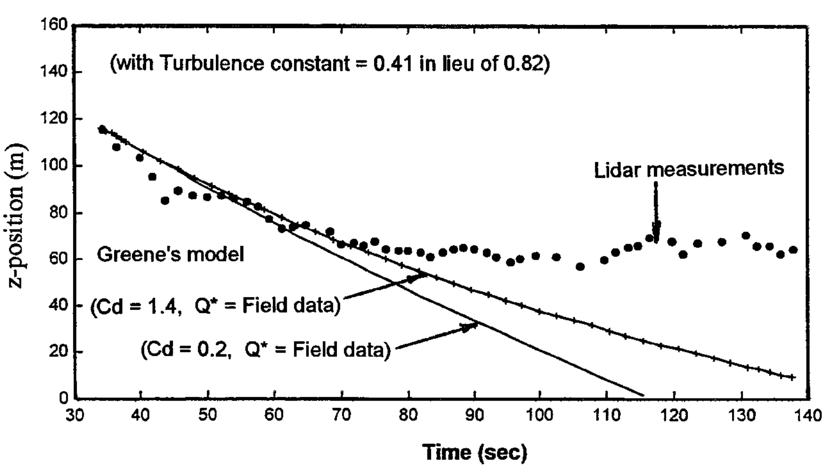
and $T^* = 9$ for all values of $\varepsilon^* < 0.001$.

The comparison of the model predictions, using $C = 0.45$, with the available field data has shown that the proposed model [Eqs. (2), (3), and (6a–6c)] agree quite well with the available data in regions out of ground effect. However, when the vortices continue to exist relatively longer times after the occurrence of a catastrophic demise event (e.g., Crow[5] instability, core bursting), the predicted circulations continue to agree with the measurements but the calculated descents lag behind that observed. This is due to the use of a constant vortex spacing even in the post-Crow period instead of an effective separation that takes into account the three-dimensional nature of the resulting vortical structures. The Crow and Bate[6] model and its modified version[15] determine the lifespan by extrapolating the linear theory to times when the displacement perturbations are comparable to the original vortex separation b. However, when a catastrophic demise event occurs, the effective vortex separation remains no longer constant and varies with the type of the structural change, time, and the ambient turbulence. For example, as the crude vortex rings in the post-Crow period undergo strong deformations, the effective vortex separation, $\Gamma/(2\pi V)$, can only be determined from the measured fall velocity and the calculated circulation from the Lidar data. It has been discovered through the evaluation of a large number of flight data that the effective separation in the post-Crow period can be expressed in terms of the maximum vortex separation minus the minimum separation divided by their sum, in a manner similar to that already done by Crow and Murman[17] and Sarpkaya.[2] Replacing the minimum separation by an effective vortex separation and simplifying, one has

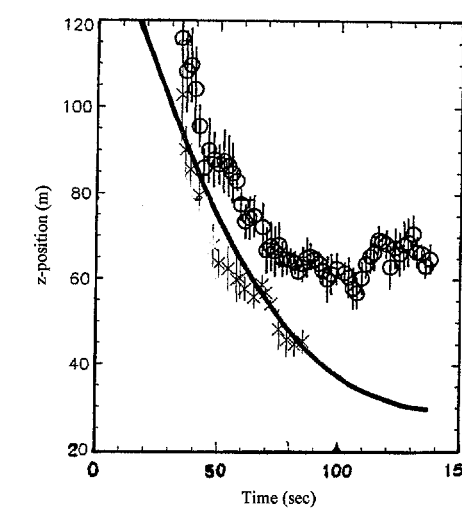
$$\frac{b}{b_0} = \frac{b(T^*)}{b_0}\left\{1 - \frac{2\alpha\varepsilon^* e/K}{1-(\alpha\varepsilon^* e)^2}\left(1 - \exp\left\langle -K\frac{T-T^*}{T^*}\right\rangle\right)\right\} \tag{7}$$

where $K = 5/\varepsilon^*$ and $\alpha = 0.5$.

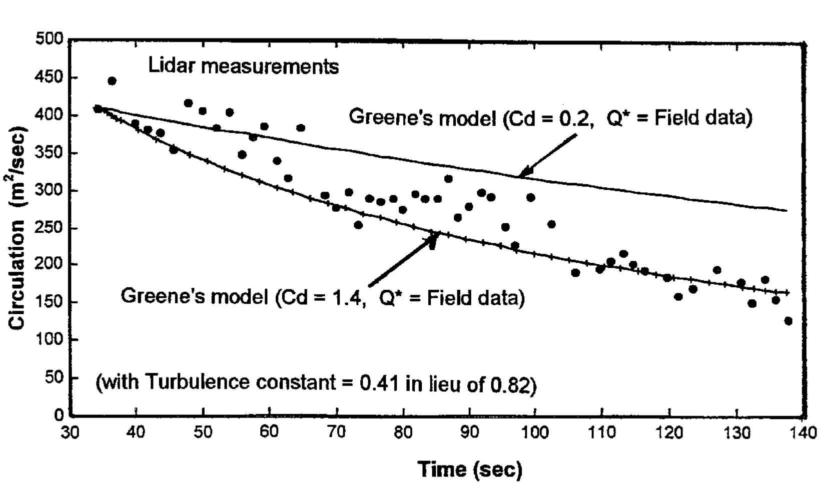
The complex problems posed by the ground effect have been studied, albeit at very low Reynolds numbers, by a number of investigators. Zheng and Ash[18] have modeled the wake vortices near ground using an unsteady, two-dimensional laminar flow. Rudis et al.[19] reported circulation measurements on wake vortices near the ground under conditions of strong stratification and wind shear. The vortices exhibited a nearly constant or slowly decaying circulation as they approached the ground. Furthermore, the path of the vortices did not exhibit the rebounding effects often seen in laminar-flow experiments and numerical simulations.[18] Robins et al.[8] devised a ground-effect algorithm to improve Greene's model by extending the earlier models (see, e.g., Corjon et al.[20]). In summary, when the vortices approach an elevation of about $2b$ from the ground, a pair of inviscid image vortices are introduced and the decay rate is assumed to remain identical to that just before the vortices entered the said region. As the elevation decreases further and reaches a level (for example, $z = b$), two new vortices (ground-effect vortices) and their images are introduced. Finally, when the second pair have rotated 180 deg around the primary vortices a second set of ground-effect vortices (and their images) are introduced. As in the image vortex region, the primary vortices (and their images) are let to decay in the ground-effect region at the rate that occurred just before the vortices entered the image vortex region. Finally, the strength of the primary vortices is reduced linearly if their elevation becomes less than $1b$. The tracking of 12 inviscid vortices in a complex ground boundary-layer environment gives the impression of a realistic simulation. However, aside from the fact that the inviscid vortices do not decay and the no-slip condition on the runway is not satisfied, the results do not appear to be commensurate with the effort.



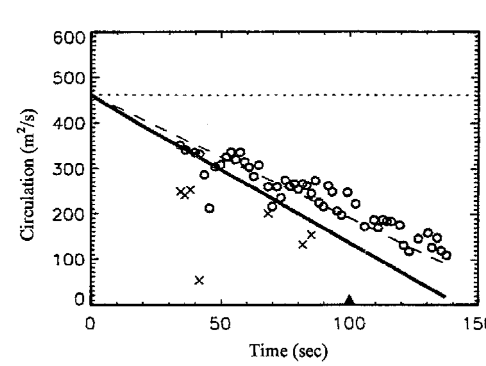
a) Elevation vs time



c) Elevation vs time with ground effect included



b) Vortex circulation vs time



d) Vortex circulation vs time with ground effect included

Fig. 2 Comparison of port vortex (M-1257) with Greene's model[16] (thick solid lines).

In our model, the ground effect is introduced in an extremely simple manner: When the two viscous vortices enter into the region $z < 1.5b$ (our definition of the ground-effect region), they continue to decay at the (constant) rate equal to that at $Z_G = 1.5$, as in Ref. 8. No other vortices, other than the primary images, are introduced into the model. The one and only change in the out-of-ground-effect region of the model is the replacement of the last term in Eq. (2) by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = -M\Gamma_0 \exp(-MT_G) = \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}T}\right)_G \tag{8}$$

where $T_G = T(Z = 1.5)$. Thus, Eq. (4) reduces to

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}T^2} + \omega^2 N^{*2} Z + M \exp(-MT_G) = 0 \tag{9}$$

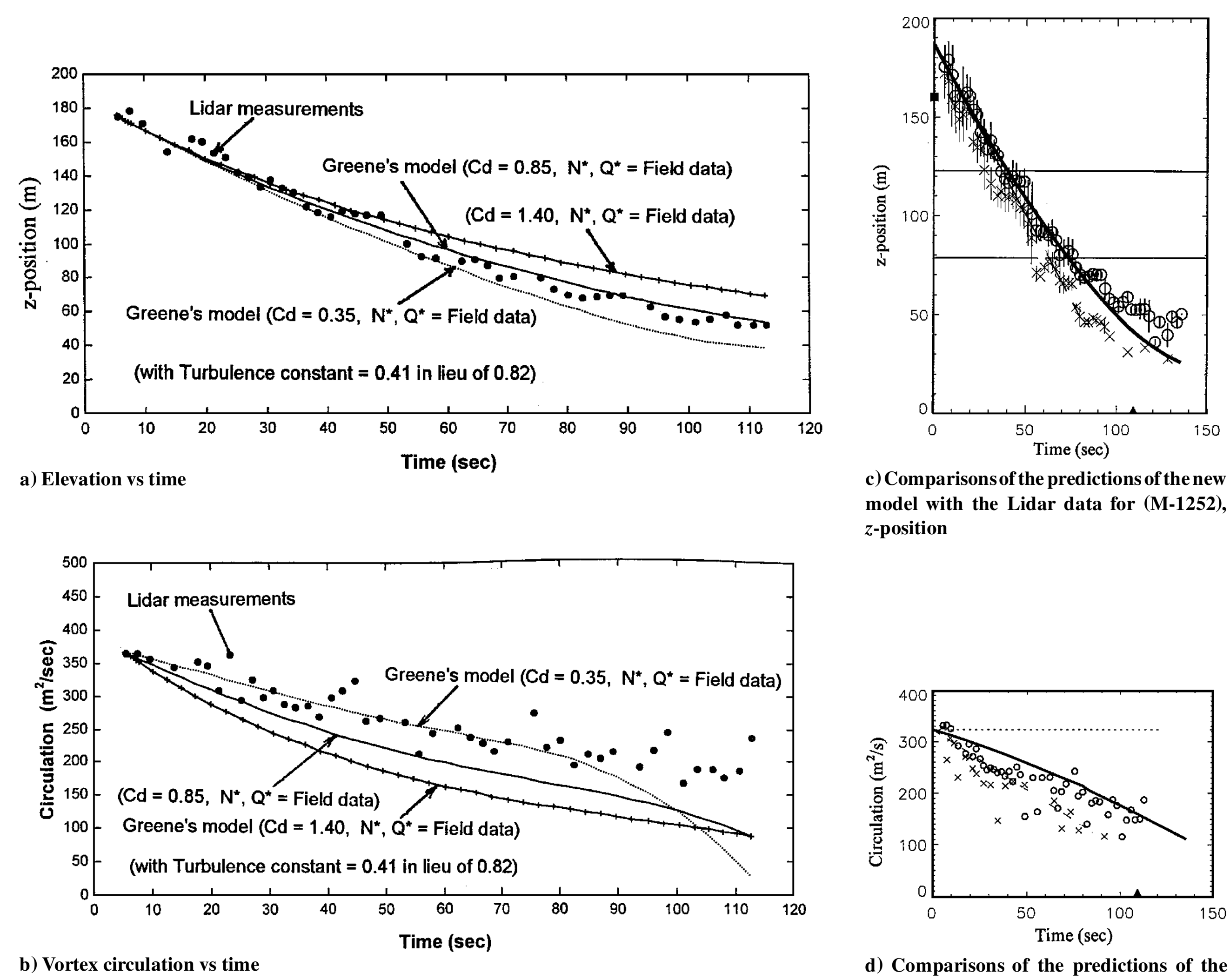
where the third term is a constant whose value is dictated by $M = C/T^*$ and T_G. The value of T^* is given by Eqs. (6a–6c) in terms of ε^*. Finally, the value of T_G is determined as part of the solution of Eq. (4). Although the model parameters have not yet been optimized, the results presented herein have been obtained using $C = 0.45$ and $Z_G = 1.5$, on the basis of preliminary calculations.

Representative Results

A large number of comparisons (mostly with flights out of the ground effect) has been made between the predictions of the new model, Greene's model,[16] and the field data from Memphis and DFW airports. Only a small fraction of these can be presented here for obvious reasons.

Figures 2a and 2b show for M-1257 the elevation and circulation data together with the predictions of the Greene model[16] for various values of the drag coefficient. Clearly, it is impossible to achieve better agreement between the calculated and measured elevations for any value of C_D from 0.2 to 1.4. However, the circulation data are well represented using $C_D = 1.4$, rather than the suggested value of 0.2 (considering the Reynolds numbers involved). Figures 2c and 2d show the same field data together with the predictions of Greene's model using $C_D = 0.2$ and the ground-effect algorithm of Robins et al.[8] Clearly, the predicted circulation with $C_D = 0.2$ does not match the data as well as Fig. 2b with $C_D = 1.4$. As another example, Figs. 3a and 3b show, for M-1252, the elevation and circulation data together with the predictions of Greene's model for various values of the drag coefficient. Figure 3a shows that it is possible to achieve a good agreement with the measured elevation with $C_D = 0.85$. However, the circulation calculations require the use of $C_D = 0.35$ for a better comparison. The purpose of these limited demonstrations is to show that the use of a drag force assumed to be acting on the Kelvin oval of a free vortex pair is not hydrodynamically sound.

Figures 3c and 3d show the predictions of the new model for the same flight (M-1252). The scatter in circulation data is usually large, as in Fig. 3d, for a number of reasons (errors in large velocities at small radii, and, in small velocities at large radii, lead to large errors in circulation at either end of the spectrum). As far as the elevation data (Fig. 3d) are concerned, the position of the port vortices (symbols ○) are more reliable than that of the starboard vortices (symbols ×), but the initial altitude is not known with sufficient precision. In other words, the predicted elevation can be shifted up or down within a band of about 20 m. For example,



a) Elevation vs time

b) Vortex circulation vs time

c) Comparisons of the predictions of the new model with the Lidar data for (M-1252), *z*-position

d) Comparisons of the predictions of the new model with the Lidar data for (M-1252), circulation

Fig. 3 Comparison of port vortex (M-1252) with Greene's model[16] (a, b) and with new model (c, d).

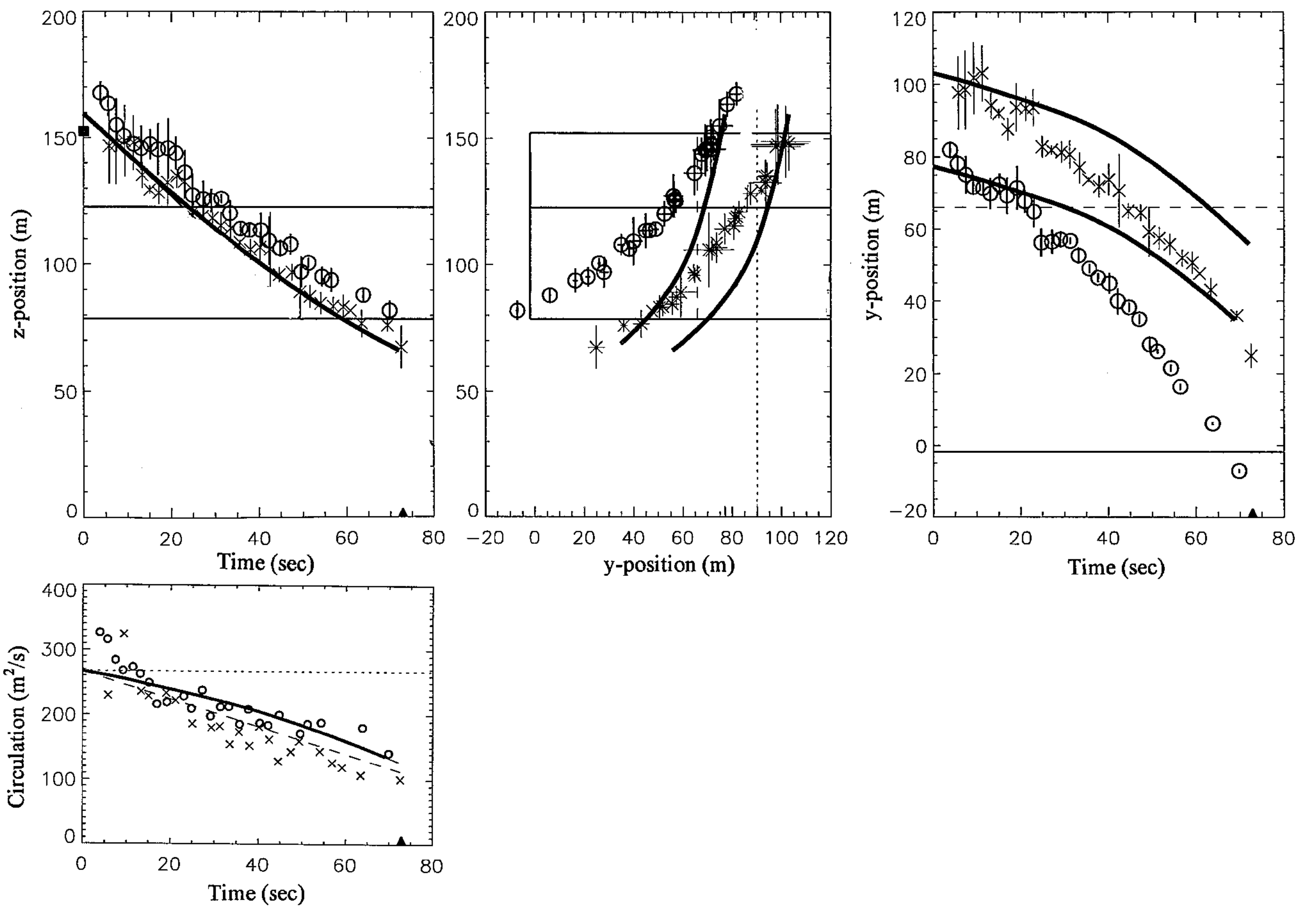


Fig. 4 Comparison of the predictions of the new model with the Lidar data for M-1291 (B-727-100): ○, port vortices, and ×, starboard.

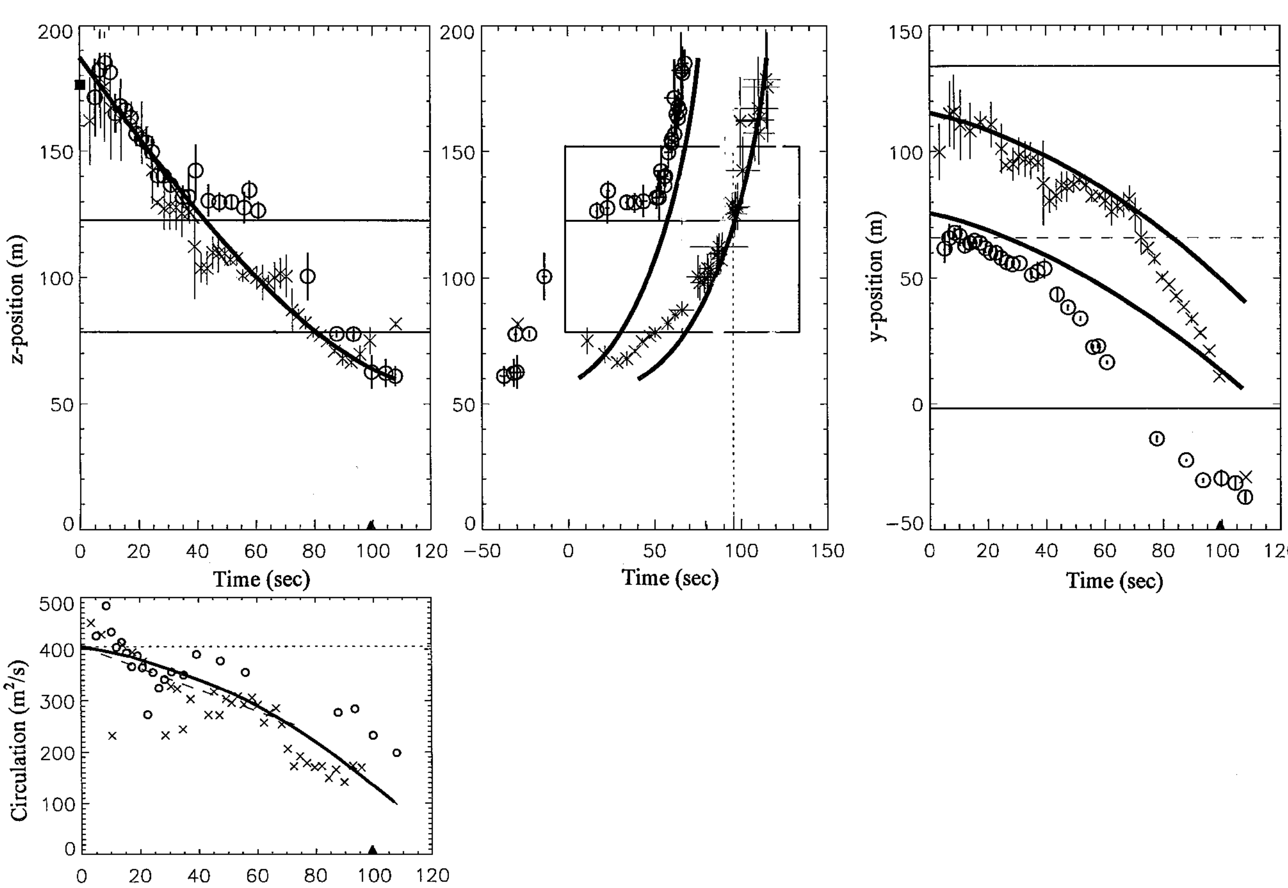


Fig. 5 Comparison of the predictions of the new model with the Lidar data for M-1284 (DC-10): ○, port vortices, and ×, starboard.

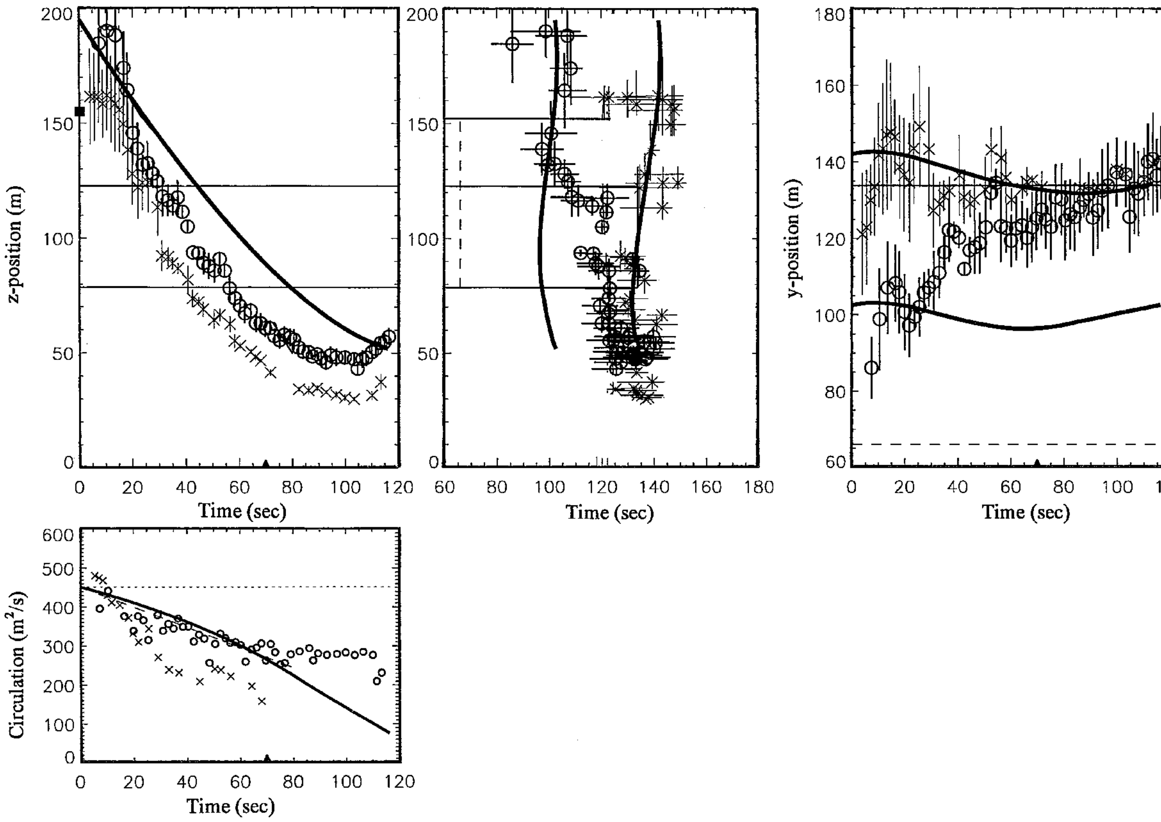


Fig. 6 Comparison of the predictions of the new model with the Lidar data for M-1270 (DC-10-30): ○, port vortices, and ×, starboard.

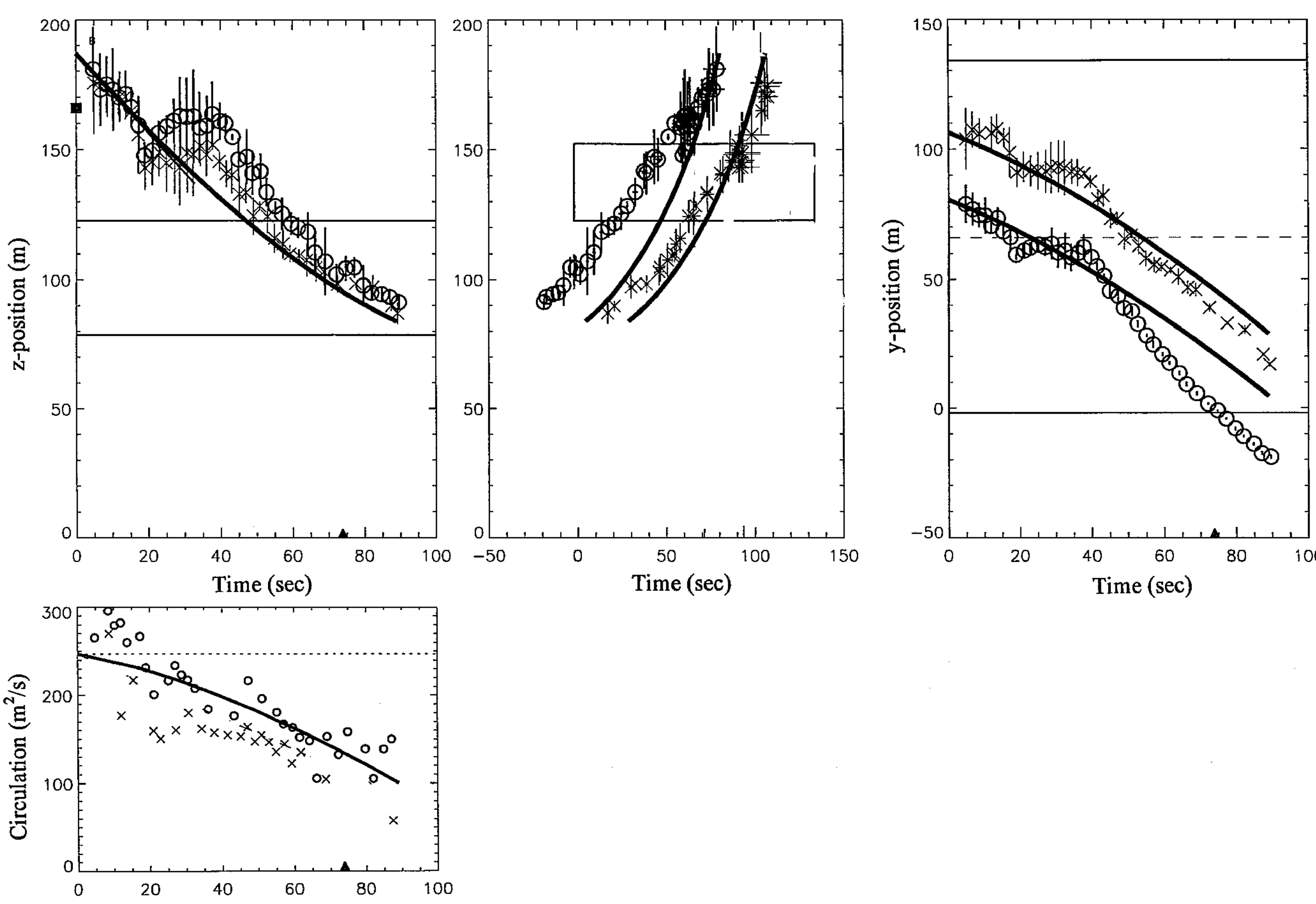


Fig. 7 Comparison of the predictions of the new model with the Lidar data for M-1170 (EA-320): ○, port vortices, and ×, starboard.

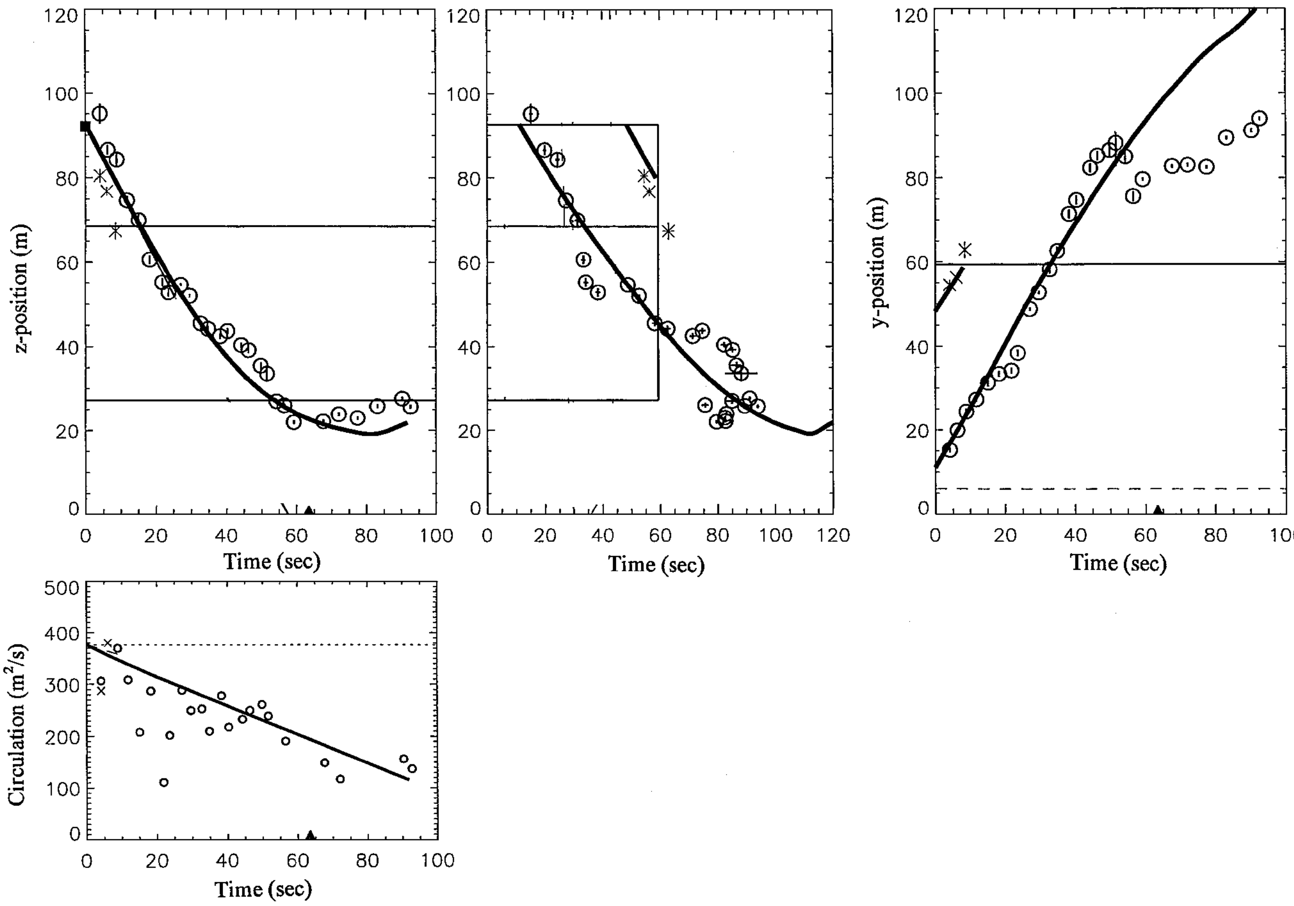


Fig. 8 Comparison of the predictions of the new model with the Lidar data for DFW-141001 (L-1011): ○, port vortices, and ×, starboard.

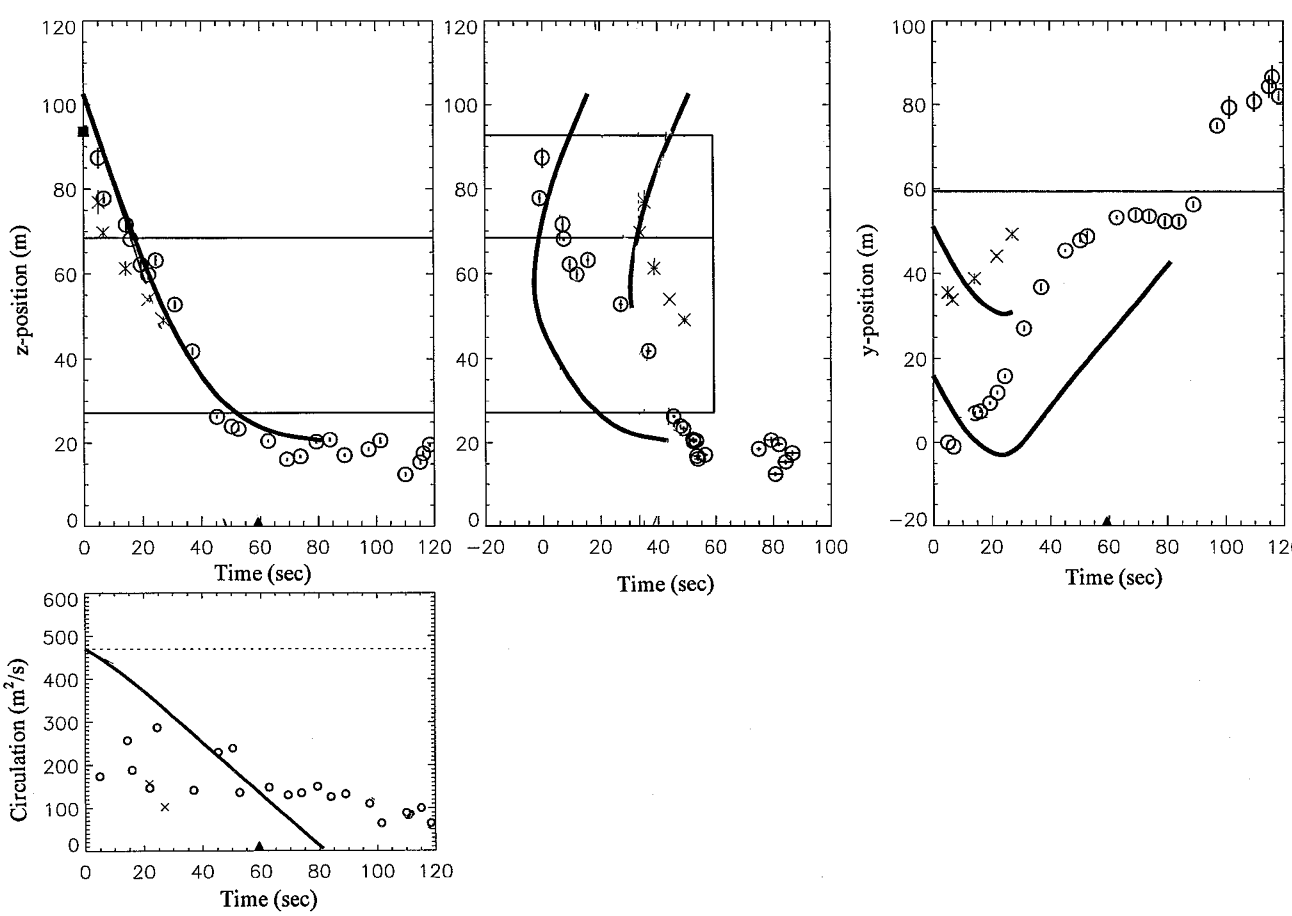


Fig. 9 Comparison of the predictions of the new model with the Lidar data for DFW-115703 (B-747): ○, port vortices, and ×, starboard.

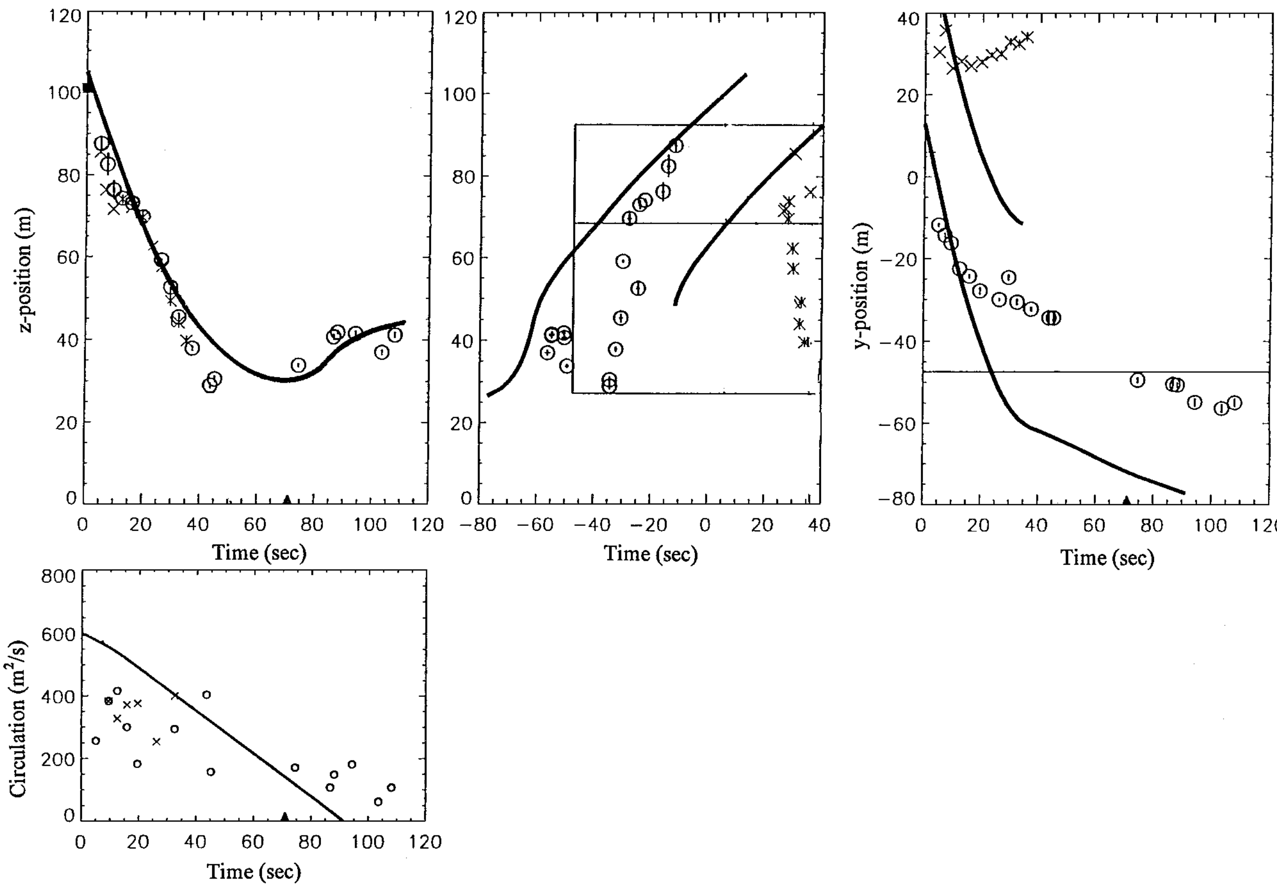


Fig. 10 Comparison of the predictions of the new model with the Lidar data for DFW-141001 (L-1011): ○, port vortices, and ×, starboard.

the shifting of the predicted line (solid curve) in Fig. 3c by about 10 m will fit the port-vortex data better. In any case, the state of the art is not sufficiently refined to deal with these second-order corrections.

Figures 4 and 5 show that the elevations and circulations are well predicted. Figures 5–7 show the comparisons of various Memphis data with the predictions of the new model. However, the predicted lateral positions of the vortex pair (z–y and y–t plots) are not sufficiently accurate. There are a number of reasons for this, the most important ones being the imprecise meteorological input on stratification, turbulence, crosswind, headwind, gravity currents, and their gradients. This is particularly important in stably stratified environments where the shear is usually strong. As noted earlier, the wind data comes only from tower measurements of 5 and 40 m, whereas the flights take place in a vertical region from about 20 to 200 m.

Figures 6 and 7 show that the new model predicts the circulations accurately, but fails to adequately represent the elevation. In the larger data set, there are a few such flights that deserve some explanation. The predicted elevation in Fig. 6 would have better agreed with the data had a starting elevation of 180 m rather than 210 m been used in the calculations. Clearly, the initial elevation data (the first five points of the port vortex) are not in conformity with the rest of the points. In fact, it appears as if the vortex pair just fell down vertically a distance of about 30 m after its inception. The error in Lidar measurements or some other natural cause cannot be ruled out. The lateral positions shown in Figs. 6 and 7 are also far from satisfactory. Note that the predictions of the Green model (not shown here) are in no better agreement with the measurements, even after considerable optimization.

Figures 8–10 show the comparison of the predictions of the new model with the data obtained at DFW airport. As in Figs. 8–10, the majority of the elevation and circulation data are well represented by the model. However, the prediction of the lateral positions and the acquisition of the necessary data toward that end remain as important challenges.

Conclusions

In summary, a relatively simple new vortex decay model has been devised. It does not violate any hydrodynamical principles, has only one model constant ($C = 0.45$), uses the eddy dissipation rate in conjunction with a theoretical model (as verified by experiments and numerical simulations), and accounts for the effective spacing of the vortices following a catastrophic demise event.

However, much work remains to be done before the model can be used for AVOSS. The analysis of the ground effect is in its preliminary stages and requires considerable additional effort. Also needed are more detailed field data (vortex velocities and positions; wind, gravity waves, and their gradients; and better temperature, humidity, and eddy dissipation profiles), the quantification and inclusion into the model of the consequences of unstable stratification, the optimization of the new model in light of better understanding of the meteorological conditions, and the differences between the wakes of various aircraft.

Most of the ambient turbulence information comes from towers about 40 m high.[7] Although, it is theoretically possible to obtain an estimate of the spectral energy density (using tower measurements of 5 and 40 m) and thereby calculate the dissipation rate directly, one obtains only an average over a height of 40 m. What is needed is the eddy dissipation from the near ground level to several hundreds of meters up to provide reliable input to the numerical simulations. Currently, this is an unresolved issue. There is no information on axial velocities in the vortices, and the tangential velocities need to be evaluated carefully.

Acknowledgments

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